Expert Systems with Applications 42 (2015) 503–509

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Texture characterization based on the Kolmogorov–Smirnov distance

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ARTICLE INFO

Article history:
Available online 23 August 2014

Keywords:
Numerical descriptor of the texture
KS statistical distance
Data mining
Image recognition

ABSTRACT

The paper proposes the new numerical descriptor of the texture based on the Kolmogorov–Smirnov (KS) statistical distance. In this approach to feature generation we consider the distribution of the pixel intensity placed in equal circular distances from the central point. In this statistical analysis each pixel of the image takes the role of the central point and KS statistics is estimated for the whole image. We determine the KS distance of pixel intensity corresponding to the coaxial rings of the increasing distance from the center. The slope of the linear regression function applied for approximating the characteristics presenting KS distance versus the geometrical distance of these rings, forms the proposed statistical descriptor of the image. We show the application of this numerical description for recognition of the set of images of soil of different type and show that it behaves very well as the diagnostic feature, better than texture Haralick features.

1. Introduction

Texture analysis and characterization form an active research topic in computer vision and pattern recognition. The recognition of texture images requires describing them by the set of numerical descriptors, well discriminating classes. These descriptors are responsible for characterizing the properties of the image and computing their similarities. In other words, they make it possible to rank texture images according to their visual properties (Gonzalez & Woods, 2011; Wagner, 1999).

Among many known methods of texture characterization to the most known belong the co-occurrence matrices, Markov random field models, wavelet and Fourier transformations, as well as local binary pattern and its variations (Chen & Kundu, 1994; Nanni, Lumini, & Brahnam, 2012; Ojala, Pietikainen, & Maenpaa, 2002; Wagner, 1999). All these methods of texture analysis exploit the statistics of the images, however, each of them in a different way. Co-occurrence matrix is relied on the original values of pixel intensities. In wavelet and Fourier approaches the image is first transformed and the statistics is based on the transformed values of pixel intensities. Markov random field texture model is characterized by geometric structure and quantitative strength of interactions among the neighboring pixels taking into account the conditional probability of signals in the neighborhood of the pixels.

Texture descriptors of images are widely applied in different fields of research, including generation of features for recognition of medical objects (Demidenko, 2004; Ojala et al., 2002), face recognition (Ahonen, Matas, He, & Pietikainen, 2009), recognition of objects in the nature (Pauwels & Frederix, 2000), recognition of different minerals (Bianconi, Gonzalez, Fernandez, & Saetta, 2012), etc. The important problem in the task of texture characterization is assuring the image invariance to the spatial scale, orientation and grayscale properties. A number of techniques incorporating these invariances have been proposed in the mentioned papers.

The effectiveness of content-based image recognition systems are very dependent on the image descriptors that are being used. In spite of the large number of methods there is still a search for the others, which allow to expand the image characterization from the other point of view. The higher the number of independent descriptors of high quality, the better is the expected efficiency of an automatic recognition system, using them as the diagnostic features.

In this paper we propose the new numerical descriptor of the images based on the Kolmogorov–Smirnov (KS) statistical distance (Corder & Foreman, 2009; Sprent & Smeeton, 2010). Some propositions of applying this statistical measure to the medical image characterization have been already presented in (Demidenko, 2004). The other application of KS statistics to texture segmentation has been also presented in (Pauwels & Frederix, 2000). All of

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http://dx.doi.org/10.1016/j.eswa.2014.08.021
0957-4174/

them apply the KS statistics for the non-parametric density estimation of the whole images. On the basis of such analysis the classification results are based.

We propose to go step ahead and extend the KS statistics to the parametric characterization of the texture image divided into smaller sub-images. We introduce special measure based on the KS distance between different sub-regions of the image. From this point of view our proposition is resembles and forms the extension of the concept of local binary pattern and its variations (Ahonen et al., 2009; Ojala et al., 2002).

The proposed method provides the invariance to the scale, orientation and rotation of the image. We consider the distribution of the pixel intensities placed in equal circular distances from the central point. In this statistical analysis each pixel of the image takes the role of the central point. The KS statistics regarding the coaxial rings around it are estimated. On the basis of such analysis we determine the KS distance of pixel intensities corresponding to the coaxial rings of the increasing distance from the center. The slope of the linear regression applied to approximate the characteristics presenting KS distance versus the geometrical distance between these rings, forms the new statistical descriptor of the image.

To illustrate our considerations we apply the method for recognition of the texture images representing 12 different classes of soil or pebbles. We show that our proposed descriptor characterizes well the classes of the images under recognition. We compare its performance to the other descriptors following from the well known Haralick gray-level co-occurrence matrices approach (Wagner, 1999) and found it competitive. In our opinion the proposed descriptor may be successfully used as the diagnostic feature in the process of automatic recognition of the texture images.

2. The basic idea of the method

Let us assume we observe one chosen point of the image and the distribution of pixel intensities around this point at increasing distance in coaxial rings. In particular we are interested in intensities of pixels placed in the rings of the increasing geometrical distances from the central point. Such division of the images into coaxial rings of the width equal 3 pixels is illustrated in Fig. 1. The central point in our analysis will travel along all pixels of the image and the results of the statistical analyses will be combined together by concatenating the pixel intensities corresponding to the same rings placed in equal distances, at different positions of the central pixel.

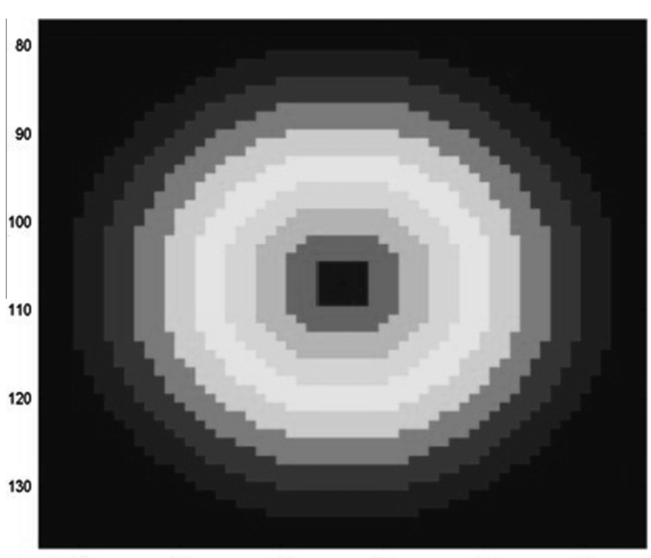

Fig. 1. The illustration of the division of the image into the coaxial rings around the central point.

In the analysis we estimate the cumulative Kolmogorov–Smirnov distances (Corder & Foreman, 2009) between the intensity of pixels x_i and x_j (samples) belonging to two different rings using Kolmogorov–Smirnov test. The KS test determines if the samples are drawn from the same underlying continuous population characterized by the cumulative distributions $F(x_i)$ and $F(x_j)$. The distance between these two populations is defined by the KS test in the form

$$d_{KS} = \max|F(x_i) - F(x_j)| \quad (1)$$

over all x. This distance may be regarded as the measure of difference between the distributions of both populations.

We calculate the KS distance for all combinations of two rings. As a result of it we get a set of KS statistics corresponding to different levels of such differences. Level 1 collects the results corresponding to KS differences of the neighboring rings, i.e., rings 1 and 2, 2 and 3, 3 and 4, etc. Level 2 corresponds to the KS differences of rings distant by 2, for example 1 and 3, 2 and 4, 3 and 5 etc. In this way we collect the KS distances corresponding to the same differences of rings for each level.

Let us take the exemplary image of the soil (bentonite) presented in Fig. 2. We have analyzed it by assuming the width of rings equal 3 pixels and by changing the positions of central point of rings, placed in all pixels of the image. To make the objective comparison of the rings of different sizes we have limited the number of pixels in the estimation process. To make the choice even most objective we perform 1000 runs at random choice of the group of 25 pixels from the ring areas (25,000 pixels totally taken into account in the statistics). The number 25 follows from the limited number of pixels in the rings close to the central point. The 1000 repetitions of the set of 25 points in estimation of KS distance have provided the reliable statistics. On the basis of many introductory experiments performed for different types of texture we have found that such choice results in a credible estimation of KS statistics. In further experiments we have kept this population of pixels unchanged.

Fig. 3 presents the exemplary results in the form of histograms of the values of KS distances for all possible combinations of rings for the first, second, third and fourth levels. It is seen that the distributions of values of KS distance change for different levels. There are clearly visible changes of the shape of histograms at different

Fig. 2. The analysed image of one particular type of soil (bentonite).

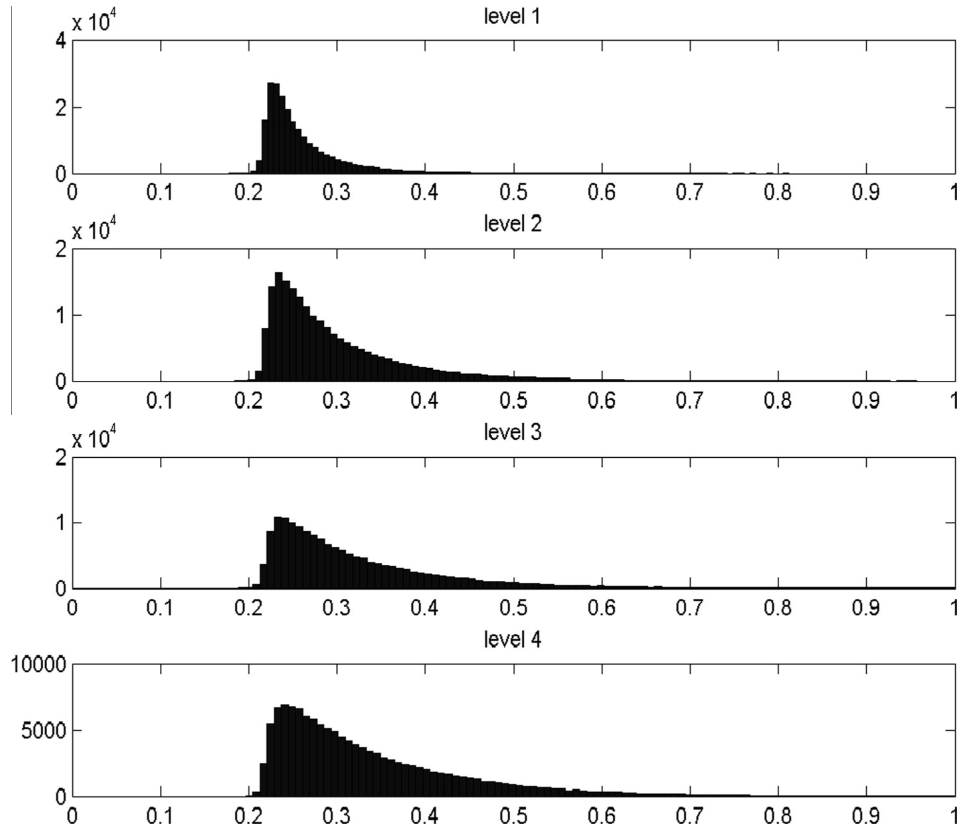

Fig. 3. The histograms of the KS distance values for the levels 1, 2, 3 and 4. The horizontal axis represents the KS distance.

levels (the higher the level the longer are the tails of the distribution). Detailed analyses performed for many other texture images have confirmed, that there is an increase of the average value of these distances at growing levels.

The next step in our approach is applying the linear regression to the relationship of the average KS distance d_{KS} as the function of the level l. In particular, we have applied the regression to the logarithmic relation presented in the form

$$\log_2(d_{KS}) = a_0 + a_1 l + \varepsilon \quad (2)$$

The coefficients a_0 and a_1 represent the estimated variables, while ε is an error of approximation. We are interested in the slope a_1 which is the most characteristic KS descriptor of the image. To limit the influence to the outliers we have used the robust regression procedure (for example in Matlab program it is a function *robustfit.m* (Matlab, 2012). The robust regression is a modification of the least squares (LS) approach and is resistive to the outliers. The LS in this approach is defined as

$$\min_{a_0,a_1} E = \sum_{l_j}\sum_{i=1}^{m_j} w_{ij}\left(\log_2\left(d_{KS_i}(l_j)\right) - y_i(l_j)\right)^2 \quad (3)$$

in which l_j represents the succeeding levels of differences ($l_j = 1, 2, \ldots$), $d_{KSi}(l_j)$ corresponds to the known ith sample (the average value of d_{KSi} on the level l_j) and y_i to the actually fitted variable $y_i(j) = a_1 l_j + a_0$. Each observed target is associated with different weight w_{ij}, adapted at each iteration in a way to reduce the influence of the actual outliers on the summed squared error. In our experiments we have used the well known Huber function (Matlab, 2012)

$$w_{ij} = \left|\frac{1}{\log_2\left(d_{KS_i}(l_j)\right) - y_i(l_j)}\right| \quad (4)$$

Fig. 4 presents the graphical results of application of the linear regression to the data of soil image of Fig. 2. The measured (known) distribution of the mean values of $\log_2(d_{KS})$ for different levels l is represented by the dashed line, and its linear approximation by the solid one. The horizontal axis represents the succeeding levels l and the vertical one – the logarithm of KS distance. The linear approximating curve is a result of regression fit estimated in this case in the following form

$$\log_2(d_{KS}) = 0.0839l - 2.0228$$

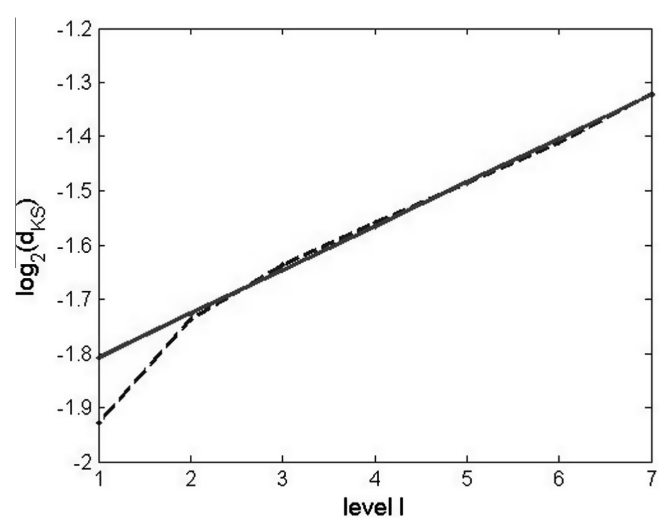

Fig. 4. The linear fit of the relationship of KS distance (logarithmic scale) versus the levels of differences between the rings.

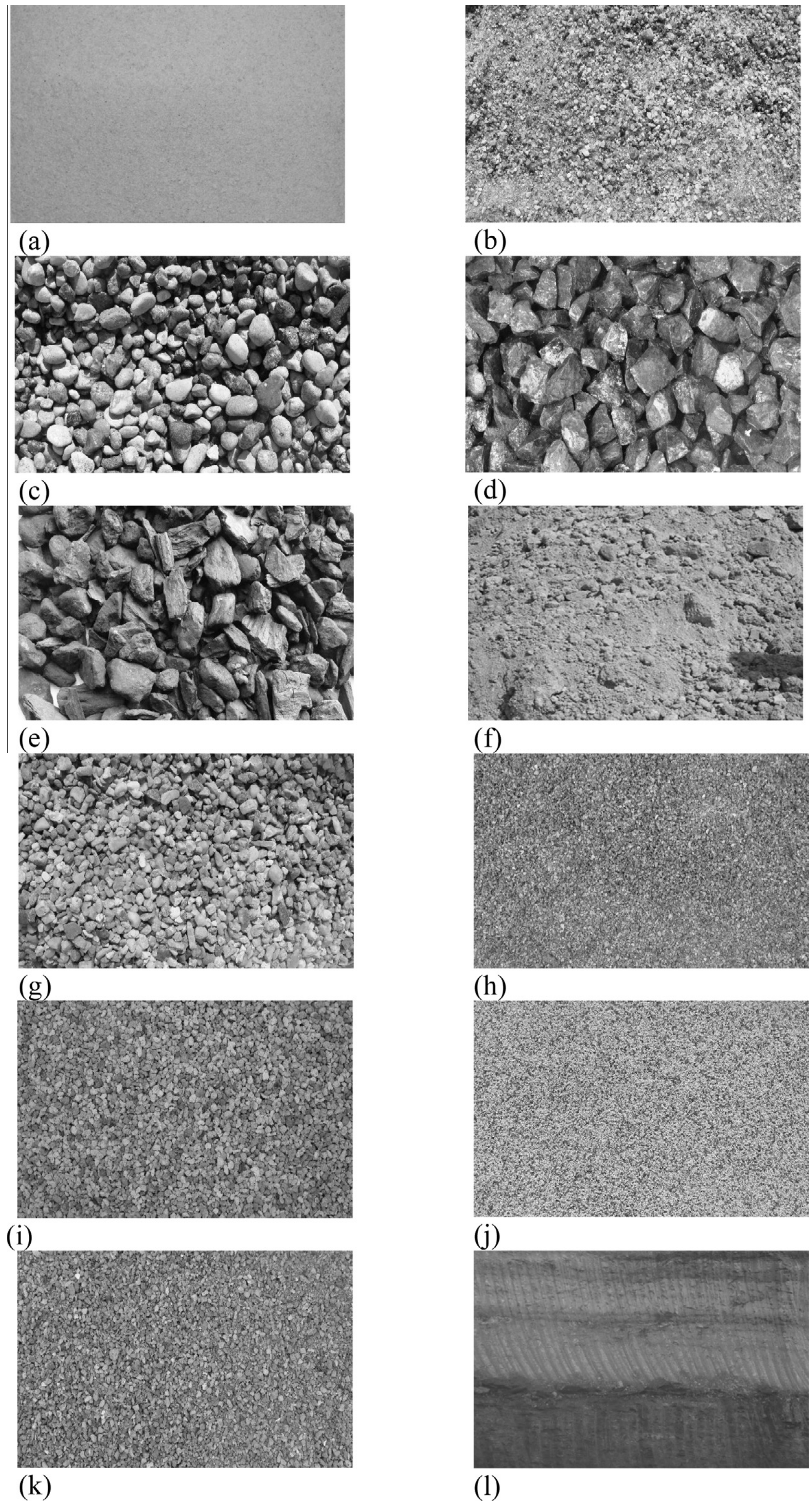

Fig. 5. The images of the analyzed classes of soil.

We can observe very good agreement of both curves starting from the second level. The parameter a_1 (the slope of the approximating linear relation) was estimated in this case as $a_1 = 0.0839$.

The slope a_1 of this relation represents the proposed KS descriptor of the image. As we will see later its value is characteristic for different types of images and may be used as the diagnostic feature for discriminating among different classes.

Table 1
The average values of the descriptor a_1 and its standard deviations for 12 classes of the analysed soil images.

Class	Mean(a_1)	Std (a_1)
a	0.0519	0.0064
b	0.0727	0.0092
c	0.1755	0.0051
d	0.1450	0.0089
e	0.1440	0.0081
f	0.0845	0.0149
g	0.1591	0.0083
h	0.0410	0.0023
i	0.0918	0.0044
j	0.0277	0.0021
k	0.0652	0.0061
l	0.1057	0.0113

The coefficient a_0 of the linear equation (the constant term) was relatively high and equal -2.0228. This value also changes with a type of the image and its discriminative ability is also worth of studying in the future.

3. The results of numerical experiments

To check the class discrimination ability of the proposed numerical KS descriptor a_1 we performed the experiments concerning the recognition between different classes of soils. Twelve classes have been considered in these experiments. They represent different types of soils and grounds existing in the nature. Their representative images are presented in Fig. 5. Some of them are very similar (for example d and e, i and k) and hence very difficult in recognition.

The image of all classes were divided into 16 smaller subimages of an identical size. In this way each class of soil was represented by 16 samples (192 analyzed samples altogether). Table 1 presents the average values of the descriptor a_1 and standard deviations for each class of soils. As it is seen the mean values differ for different classes. On the other hand the values of a_1 for similar images (for example class d and e) are very alike (the negligible differences in the means and standard deviations). In each case we observe very small values of the standard deviations. This is very good prognostic for its application as a descriptor of the image.

To assess the discriminative ability of the proposed descriptor we have determined its Fisher discriminative measure, defined in the way (Duda, Hart, & Stork, 2003; Tan, Steinbach, & Kumar, 2006)

$$S_{AB}(a_1) = \frac{|c_A(a_1) - c_B(a_1)|}{\sigma_A(a_1) + \sigma_B(a_1)} \qquad (5)$$

In this definition c_A and c_B are the mean values of the feature a_1 for the images of the class A and B, respectively. The variables σ_A and σ_B represent the standard deviations determined for both classes. The large value of $S_{AB}(a_1)$ indicates good potential separation ability of the feature a_1 for the classes A and B. On the other side small value of it means that this particular feature is not good for the recognition between these classes. Table 2 presents the values of the Fisher discrimination measure for all 66 two-class combinations arranged among 12 considered classes.

As we can see the Fisher discriminative measure of the descriptor assumes high values for most combinations of classes. The only exceptions represent classes d and e, (the discriminative measure equal 0.05), which correspond to the images, which were very similar to each other.

To check the real performance of the KS descriptor in image recognition process we have analysed its class discriminating ability by creating the ROC (Receiver Operating Characteristic) curves (Tan et al., 2006) for all combinations of 2 classes (one positive class and the second, treated as the negative one). The ROC curve is a graphical approach for displaying the trade-off between true

Table 2
The values of Fisher discriminant measure for recognition between all combinations of classes.

Classes	a	b	c	d	e	f	g	h	i	j	k	l
a		1.33	10.73	6.05	6.35	1.53	7.26	1.25	3.68	2.83	1.06	3.03
b	1.33		7.21	3.99	4.14	0.49	4.94	2.76	1.41	3.98	0.49	1.61
c	10.73	7.21		2.17	2.39	4.54	1.22	18.24	8.78	20.52	9.89	4.25
d	6.05	3.99	2.17		0.05	2.53	0.82	9.25	3.97	10.60	5.31	1.94
e	6.35	4.14	2.39	0.05		2.58	0.91	9.94	4.17	11.41	5.57	1.97
f	1.53	0.49	4.54	2.53	2.58		3.20	2.53	0.37	3.33	0.92	0.80
g	7.26	4.94	1.22	0.82	0.91	3.20		11.12	5.27	12.58	6.52	2.72
h	1.25	2.76	18.24	9.25	9.94	2.53	11.12		7.57	3.02	2.89	4.76
i	3.68	1.41	8.78	3.97	4.17	0.37	5.27	7.57		9.80	2.54	0.88
j	2.83	3.99	20.52	10.60	11.41	3.33	12.58	3.02	9.80		4.58	5.81
k	1.06	0.49	9.89	5.31	5.57	0.92	6.52	2.89	2.54	4.58		2.33
l	3.03	1.61	4.25	1.94	1.97	0.80	2.72	4.76	0.88	5.81	2.33	

Table 3
The *AR* values corresponding to the recognition of all pairs of classes at recognition of 12 classes of soil on the basis of only descriptor a_1.

	a	b	c	d	e	f	g	h	i	j	k	l
a	0	0.94	1	1	1	0.98	1	0.82	1	1	0.87	1
b	0.94	0	1	1	1	0.52	1	1	0.96	1	0.49	0.99
c	1	1	0	1	1	1	0.95	1	1	1	1	1
d	1	1	1	0	0.08	1	0.72	1	1	1	1	0.99
e	1	1	1	0.08	0	1	0.81	1	1	1	1	0.99
f	0.98	0.52	1	1	1	0	1	1	0.45	1	0.77	0.78
g	1	1	0.95	0.72	0.80	1	0	1	1	1	1	1
h	0.82	1	1	1	1	1	1	0	1	1	1	1
i	1	0.96	1	1	1	0.45	1	1	0	1	1	0.76
j	1	1	1	1	1	1	1	1	1	0	1	1
k	0.87	0.49	1	1	1	0.77	1	1	1	1	0	1
l	1	0.99	1	0.99	0.99	0.78	1	1	0.76	1	1	0

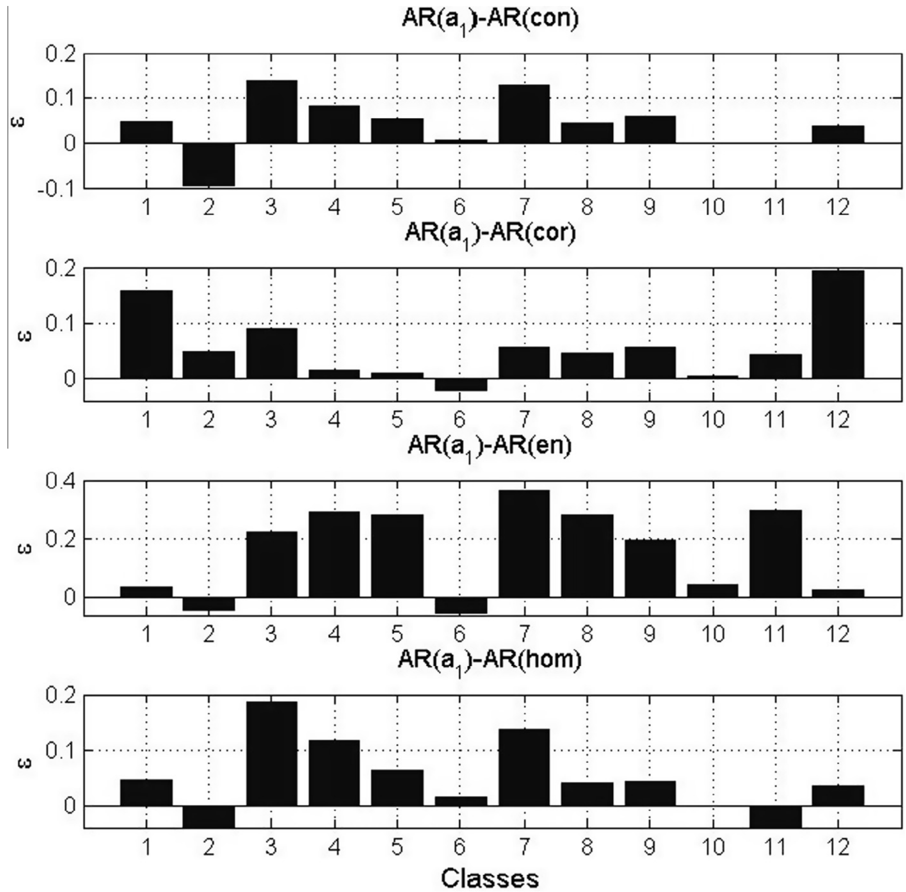

Fig. 6. The distribution of mean differences between the values of *AR* corresponding to the KS descriptor and 4 Haralick descriptors: (a) contrast (*con*), (b) correlation (*cor*), (c) energy (*en*) and (d) homogeneity (*hom*) for recognized classes.

positive rate (TPR) and false positive rate (FPR) of a classifier, based on the particular feature (Tan et al., 2006). In ROC curve TPR is plotted along the *y* axis and FPR is shown on *x* axis. The point (TPR = 0, FPR = 0) corresponds to the situation, when model predicts every instance to be a negative class. The point (TPR = 1, FPR = 1) represents the case, when model predicts every instance to be a positive class, and (TPR = 1, FPR = 0) represents an ideal model. Good classification model is located as close as possible to the upper left corner of the diagram. On the other hand the model that makes random guesses resides along the main diagonal, connecting points (TPR = 0, FPR = 0) and (TPR = 1, FPR = 1).

The quality of a classifier is measured by the area under curve (AUC). AUC can be interpreted as an average ability of the model for classifying observations accurately into two classes (Tan et al., 2006). A higher area denotes higher discrimination ability. If the model is perfect then the AUC is equal 1. If the model performs like a random guess then the AUC is equal 0.5. The model better than random should have area larger than 0.5. The overall advantage of the investigated classifier model over the random one is characterized in the form of the accuracy rate (*AR*) defined as follows

$$AR = 2 \times \text{AUC} - 1 \tag{6}$$

AR is the summary index of cumulative accuracy profile (also known as Gini coefficient). It is interpreted as the ratio of the area between the AUC of the rating model being validated and the AUC of the random model. It shows the performance of the evaluated model by depicting the percentage of right scores provided by the model across different scores. The value *AR* = 0 means that our classifier is equivalent to the random one. Any positive value of it indicates advantage of the evaluated model over the random one. The value 1 means ideal classification performance of the model. Table 3 presents the values of *AR* of our classification model applying only the feature a_1 in the task of recognition of all pairs of classes among 192 images representing 12 classes of soils (16 subimages of each class) presented in Fig. 5.

As we can see the classification model based on the proposed KS descriptor is always better than the random one. For most combinations of 2-classes its performance is ideal (perfect recognition of classes, reflected by the value *AR* = 1). There is only one small value of *AR* = 0.08 corresponding to the recognition between very similar classes of pebbles (classes d and e). The average value of *AR* calculated over all pairs of images was equal 0.94.

4. Comparison to Haralick texture descriptors

The developed descriptor is based in practice on the statistics of the group of pixels intensities in coaxial rings, and hence represents some counterpart to the texture Haralick descriptors based on the so called gray-level co-occurrence matrices (Wagner, 1999). The co-occurrence matrices (the square matrices with dimension *N*, where *N* is the total number of gray levels in the image) focus on the distribution and the relationships among the gray levels of the neighboring pixels of the image. The [i, *j*]th element of this matrix is produced by counting the total occasions a pixel with value *i* is adjacent to a pixel with value *j* and then subsequently dividing the whole matrix by the total number of such comparisons that are made. The adjacency can be defined to take place in each of four directions in a two-dimensional square pixel image (horizontal, vertical, left and right diagonals) and as a result we can calculate four such matrices. The texture features are calculated by averaging over all these four co-occurrences matrices.

From the co-occurrence matrices many Haralick texture descriptors of statistical nature can be evaluated. In this comparison we will limit our consideration to only four: contrast (the local contrast in an image) denoted as *con*, correlation (a correlation of pixel pairs on gray-levels) denoted as *cor*, energy (the occurrence of repeated pairs within an image) denoted as *en* and homogeneity denoted as *hom* (coefficient characterizing the distribution of the elements of a GLCM with its diagonal, which assumes value between zero and one).

We have created the differences of *AR* measures corresponding to the proposed KS descriptor and to the appropriate values representing these four Haralick descriptors for all combinations of two classes of soils presented in Fig. 5. To determine the global measure of the feature quality for each recognized class we have calculated the mean value of the difference $AR(a_1) - AR(Haralick)$, where the *Haralick* was one of four parameters: *con, cor, en* and *hom*.

Fig. 6 shows the plot of the differences $\varepsilon = mean(AR(a_1) - AR(Haralick))$ between the values of *AR* corresponding to our KS descriptor and the texture Haralick descriptors. They have been calculated for the recognition of all 16 representatives of the particular class from the other classes of soils. The succeeding classes from 1 to 12 correspond to the texture images denoted in Fig. 5 by the letters from a to l, respectively. The horizontal axis represents the succeeding class and the vertical one – the appropriate mean value of these differences. We can easily note, that for most classes the performance of our descriptor as a diagnostic feature is better than any of the Haralick descriptors (the positive values of the differences).

Except one or two classes presented in each row the discrimination value of our descriptor was better than the traditional Haralick descriptors. The average value of the *AR* measure of the proposed KS descriptor for combination of all classes was equal 0.938. The appropriate values for Haralick descriptors were $AR(con) = 0.896$, $AR(cor) = 0.882$, $AR(en) = 0.776$, $AR(hom) = 0.899$.

5. Conclusions

The paper has presented the novel approach to the generation of the numerical descriptors of the texture images. It is based on the application of Kolmogorov–Smirnov statistical distance applied to the pixel intensities in the coaxial rings formed for all possible positions of the central pixel in the image. Instead of applying the KS statistics to the whole image we divide the image into many coaxial rings and apply this statistics to all combinations of rings, placed in different sub-regions of the image. Thanks to this we are able to propose the parametric characterization of the image, which is better suited for finding the differences between the sub-images belonging to different classes. Our proposition applies linear regression to the curve representing the KS distances between the pixel intensities of the succeeding rings versus different levels of geometrical differences between the rings. The slope of the regression curve represents the proposed descriptor.

We have compared our approach to the well-known texture Haralick descriptors. Both approaches use the statistical information contained in the mutual interrelations between groups of pixels. However, our proposed characterization of the image is of different nature than that of Haralick. Instead of direct interrelations between pixels in the close neighborhood we apply more general information contained in the pixels characterized by the KS statistical measure. In contrast to the co-occurrence approach or local binary patterns our measure is blind to direction of processing in a two-dimensional square pixel image. On the basis of the numerical experiments concerning the images of 12 classes of soils we have found that our descriptor is of better class discrimination ability than the well-known Haralick descriptors. The only weakness of the proposed approach is a bit longer time of calculations. The experiments comparing our method with Haralick approach have shown higher time consumption (around 30% higher) in the analysis of the same set of pictures.

Our approach to feature generation does not mean that the proposed descriptor should replace any of the existing ones used in texture characterization. Because of different nature of generation it is ideally suited for cooperation with the set of the others. Many independent features acting collectively increase the probability of achieving better classification accuracy in the texture recognition process.

For future studies, we plan to investigate also the discrimination ability of the second coefficient a_0 of the linear regression or another form of regression. On the basis of the introductory experiments we expect that such approaches may also return the promising results. In the future we will also try applying our approach for another types of images. Especially interesting are the images of biomedical objects. The next point of research is to study the cooperation of our descriptor with the descriptors, which are generated using other methods. This should allow to get even better results of image characterization.

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